# SOLO and solid geometry: Interpreting upper primary students' representations of the three-dimensional geometrical shapes and their development of three-dimensional visualization

by

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A thesis submitted for the Degree of
Doctor of Education

Faculty of Arts and Social Sciences
University of Technology, Sydney

2009

## Certificate of Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis have been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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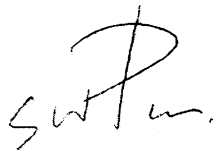

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## Acknowledgement

I would like to give special thanks to many people who provided encouragement, help as well as challenging questions during the development of the study and the writing of this thesis.

My heartfelt thanks to my supervisor, Dr Tony Holland. Tony's expertise, interest and insights in this research, as well as his comments and questions in a "wait-and-see" ways after my dark times, have been vital in the conceptualisation and completion of this study.

I particularly thank Dr Alison Lee whose challenging questions helped me to reorient the research focus. My thanks to all the professors in Faculty of Education who helped to offer and/or conducted courses in this EdD programme. I could not formulate the research framework and the theoretical background of this research without these courses.

My very special thanks go to the principals, teachers and students in the three schools where the data was collected. I could not have done the study without them.

Finally, this thesis could not be done without the support and critical comments from other supervision panel members: Dr Diana Slade, Dr Bob Pithers and Dr Kitty te Riele.

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## Abstract

Within the study of the three-dimensional (3D) geometry in the primary school curriculum is the development of students' spatial visualization, the ability to visualize the 3D geometrical shapes. Both the National Council of Teachers of Mathematics in the States and the Curriculum Development Council in Hong Kong emphasis that all students should have opportunities to visualize and work with 3D shapes in order to develop spatial skills fundamental to everyday life and to many careers.

This research is a study about how a sample of 297 upper primary students in Hong Kong describe in (Chinese) words about, and draw on papers, the 3D geometrical shapes. These shapes include cube, triangular prism, pyramid and cylinder. The focus is on the interpretation of the diversity of the responses used in students' descriptions and on the exploration of students' development of visualization by their drawings of the four 3D shapes. It raises issues about upper primary students' spatial ability and the need to conceptualize 3D spatial ability within board theoretical frameworks in the literature. The theoretical models of different aspects are explored: Bishop's IFI and VP spatial abilities be adopted to define the scopes of spatial abilities; Van Hiele's levels of geometry learning be the basis of the macro stages of cognitive development; and the Prestructural-Unistructural- Multistructural-Relational (P-U-M-R) sequence in the SOLO model be the micro levels of cognitive development growth.

Epistemologically, the view of knowledge in this study is based on Vygotsky's socialcultural perspective. The grounded theory approach and the interpretive approach are adopted for the research methodology.

A fundamental finding is that the response patterns about the descriptions and the drawings of the 3D geometrical shapes are rich and diverse. Also, the findings support the P-U-M-R levels of the SOLO model and provide a set of descriptors of students' developmental representation patterns for the four levels both in

descriptions and drawings underpinned by the theoretical framework achieved in this study.

This study concludes with recommendations for explicit instructions on the 3D geometrical shapes such as the 3D geometrical vocabulary, 3D drawing conventions, the static nature of representational drawings on papers, building up the coordination views, the partial occlusion technique, hands-on activities with real objects and relating 3D objects in the real world.

Key words: 3D geometry; spatial ability, SOLO model